

## MULTI-OBJECTIVE OPTIMIZATION OF REINFORCED CONCRETE SLABS EXPOSED TO NATURAL FIRES

Ranjit Kumar Chaudhary<sup>1</sup>, Balša Jovanović<sup>2</sup>, Thomas Thienpont<sup>3</sup>, Thomas Gernay<sup>4</sup> and Ruben Van Coile<sup>5</sup>

### ABSTRACT

Risk-based designs are complicated and often computationally challenging, involving numerous design parameters, design objectives and design constraints. In structural fire engineering, single-objective optimization is commonly adopted for risk-based designs, but this approach may converge to a local optimum, leading to uneconomical design solutions. Implementation of multi-objective optimization (MOO) algorithms can address these issues for risk-based designs. This study explores the implementation of MOO for design of reinforced concrete slabs exposed to natural fires. The natural fire exposure is modelled through the Eurocode parametric fire curve, relating the fire load density to the building occupancies (as listed in EN 1991-1-2:2002). To reduce the computational cost, a non-dominated sorting genetic algorithm (NSGA-II) is considered for MOO in this study. Adopting MOO approach, an optimized design is obtained which reduces the reinforcement cost in slab by 40 % and the environmental cost by 25 %, compared to the prescriptive design approach. The results from such risk-based design optimization could be taken into account when defining prescriptive design requirements.

**Keywords:** Reinforced concrete slab; natural fires; risk-based design; multi-objective optimization; genetic algorithm

### 1 INTRODUCTION

Fire safety regulations are commonly implemented through prescriptive design guidance. These guidance are based on the historical experience, learnt over time in response to fire disasters. In most cases, adequate safety can be achieved in (common) structures which fall within this experience [1], but adequacy for uncommon structures is unproven. With the increased use of innovative structural materials, preference of exceptional architectural designs and the adoption of advanced engineering solutions, the application of prescriptive guidance is narrowing. Realising the limitations of prescriptive guidance, performance-based design (PBD) is increasingly preferred [2]. One of the commonly accepted tools for PBD is probabilistic risk assessment (PRA) [1]. In performance-based approaches, fire safety objectives are clearly defined.

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<sup>1</sup> PhD student, Ghent University

e-mail: [RanjitKumar.Chaudhary@ugent.be](mailto:RanjitKumar.Chaudhary@ugent.be), ORCID: <https://orcid.org/0000-0002-1104-5859>

<sup>2</sup> PhD student, Ghent University,

e-mail: [Balsa.Jovanovic@ugent.be](mailto:Balsa.Jovanovic@ugent.be), ORCID: <https://orcid.org/0000-0001-5200-5848>

<sup>3</sup> PhD student, Ghent University

e-mail: [Thomas.Thienpont@ugent.be](mailto:Thomas.Thienpont@ugent.be), ORCID: <https://orcid.org/0000-0003-1466-3377>

<sup>4</sup> Assistant professor, Johns Hopkins University

e-mail: [tgernay@jhu.edu](mailto:tgernay@jhu.edu), ORCID: <https://orcid.org/0000-0002-3511-9226>

<sup>5</sup> Associate professor, Ghent University

e-mail: [Ruben.VanCoile@ugent.be](mailto:Ruben.VanCoile@ugent.be), ORCID: <https://orcid.org/0000-0002-9715-6786>

Societal objectives such as life-safety form the benchmark for the design and the stakeholders are free to introduce their additional objectives (e.g., reduced structural down-time after damage, higher design life, intact aesthetic appearance, etc.). The PRA based performance criteria for design objectives are aimed at reducing the structural risk to “As Low As Reasonably Practicable” [1,3]. The probabilistic evaluation of these criteria however demands huge computational expense.

Risk-based designs in fire safety engineering are generally single-objective optimization based, especially through cost-benefit analysis by balancing cost of a safety measure against the benefits of risk reduction [4, 5, 6, 7]. In these investigations, when multiple objectives exist, they are combined to form a single objective function. These single-objective based optimizations may converge to a local optimum, potentially leading to uneconomical results [8]. Further, for developing single-objective formulations, all the objectives need to be transformed to a single utility value. For example, when the optimization includes the minimization of CO<sub>2</sub> emissions, the objective is often transformed into an equivalent monetary value. Adoption of a multi-objective optimization (MOO) method has the potential to address these shortcomings of single-objective optimization problems [8], but has yet to be studied in the structural fire engineering context.

The current study shows that risk-based design can be carried out through multi-objective optimization. To demonstrate this, a case study of a fire exposed reinforced concrete (RC) slab is considered. The fire exposure for the slab is modelled by a natural fire, including the decay phase. For the MOO of the RC slab, a computationally efficient genetic algorithm (GA) is adopted [8]. This study therefore demonstrates a novel risk-based design approach for fire exposed structures. Within the approach, additional design objectives, design parameters, and constraints can be introduced, leading to a flexible PBD framework.

## 2 MULTI-OBJECTIVE OPTIMIZATION METHOD

The solution of a MOO problem involves determining the design variables that result in optimum objective function values. These optimum design variables lie within the feasible region. Eq. (1) represents a general MOO problem:

$$\begin{aligned}
 &\{ \text{Maximize/minimize} && f_m(x), && m = 1, 2, \dots, M \\
 &\text{subjected to} && g_j(x) \geq 0, && j = 1, 2, \dots, J; \\
 &&& h_k(x) = 0, && k = 1, 2, \dots, K; \\
 &&& x_i^L \leq x_i \leq x_i^U, && i = 1, 2, \dots, n \}
 \end{aligned} \tag{1}$$

where  $x = [x_1, x_2 \dots x_n]^T$  is a vector of design variables. The design variables themselves are subjected to the last set of constraints, with  $L$  the lower bound and  $U$  the upper bound. These bounds constitute the variable design space  $\mathcal{D}$ . Further,  $g_j(x)$  and  $h_k(x)$  are a set of  $J$  inequality and  $K$  equality constraints and  $f_m(x)$  is a set of  $M$  design objectives. The design objectives form the objective space  $\mathcal{Z}$ .

Single-objective optimizations result in a single-optimal solution (a point), while the MOO method results in a set of optimal solutions, commonly referred to as Pareto-optimal solutions. The solutions are a set of non-dominated points calculated through the principle of dominance, which states that these sets are the ones that are not dominated by any other solution set. The boundary defined by these Pareto-optimal solutions is called a Pareto-optimal front. The main objective of MOO is finding the set of solutions which are diverse (spread-over the objective space) and close to the Pareto-optimal front.

MOO is a well-recognised approach in engineering for solving a variety of problems [8]. Even in structural engineering, the application of MOO for design optimization is well-recognised. Some of the applications related to seismic design can be found in [9-11], where member cross-sections are optimized by minimizing the structural damage and cost. Other applications relate to the maintenance planning for reinforced concrete bridges [12], minimizing environmental effects of RC frame designs [13], in construction project management [14] etc. MOO algorithms can mainly be classified into two groups: (i) Classical methods and (ii) Evolutionary algorithms. The classical methods have several disadvantages: they are slow and require many computations, resulting in a biased solution. They often converge to a local/sub-optimal solution and are difficult to implement with parallel computing [8]. Most of the evolutionary algorithms overcome these

limitations. Therefore, the current study adopts a genetic algorithm (non-dominated sorting algorithm, referred as NSGA-II), which is a type of evolutionary algorithm [8-10].

### 3 REINFORCED CONCRETE SLAB EXPOSED TO NATURAL FIRES

#### 3.1 Problem description

The considered RC slab is one-way simply supported, 0.2 m thick and reinforced at the bottom face with bars of 10 mm diameter spaced at 100 mm centre to centre. The reinforcement bars have a clear concrete cover of 15 mm (i.e., the reinforcement axis at 20 mm from the bottom face). The concrete slab is a part of a compartment of size 6 m × 4 m × 3 m. The concrete has a characteristic strength of 30 MPa, while the yield strength of reinforcement is 500 MPa. The same slab has been investigated in [16, 17] for probabilistic assessment of fire-exposed structures. The slab is exposed to fire at the bottom face, while the top face is in contact with air at ambient temperature. The fire exposure to the slab is modelled using the Eurocode parametric fire curve [15]. Slabs from two types of building occupancies (office and residential building, considering fire load densities listed in the EN 1991-1-2:2002) are considered for the evaluations [15].

#### 3.2 Bending moment capacity evaluation

The bending moment capacity of the slab ( $M_R$ ) is evaluated based on Eq (2), which allows determining the bending moment capacity at both the ambient and elevated temperatures. At ambient condition, the temperature of reinforcing bars ( $T$ ) is considered as 20°C. For fire exposure, a thermal analysis is carried out to evaluate the temperature in the rebars over time. In Eq (2),  $A_s$  refers to the area of reinforcement in the slab,  $k_{f_y(T)}$  is the yield strength retention factor for reinforcement yield stress,  $h$  and  $b$  refers to the depth and width of the slab,  $c$  is the clear concrete cover to reinforcement and  $\phi$  is the diameter of reinforcement.  $f_{y,20^\circ C}$  and  $f_{c,20^\circ C}$  stand for yield strength of reinforcing bars and strength of concrete, respectively, at ambient temperature. The design bending moment capacity of the slab at ambient conditions is calculated as 59 kN-m (considering 1.5 and 1.15 as the safety factor for concrete and steel strength).

$$M_R = A_s k_{f_y(T)} f_{y,20^\circ C} \left( h - c - \frac{\phi}{2} \right) - 0.5 \frac{\left( A_s k_{f_y(T)} f_{y,20^\circ C} \right)^2}{b f_{c,20^\circ C}} \quad (2)$$

The thermal analysis for RC slab is carried out through a 1-D numerical heat transfer model. In the analysis, the entire cross-section of the slab is modelled as concrete, and the fire exposure at the bottom face is applied through convection and radiation. More details and validation of the numerical heat transfer can be found in [17]. The thermal evaluation for the RC slab is carried out until the temperatures in the section cool down close to ambient, to consider stability until full ‘burnout’ [18]. Figure 1 shows the maximum temperature of the reinforcing bars evaluated for natural fire exposure of RC slab. Herein, various parametric fires with varying fire load density ( $q_f$ ) and opening factor ( $O$ ) are considered. The results are shown for two reinforcement axis distances (20 and 60 mm). In the Figure, the temperatures vary widely for change in both the parameters of parametric fire exposure ( $q_f$  and  $O$ ) and the reinforcement axis distances. With the maximum temperature of reinforcing bars determined, the critical bending moment capacity of RC slab for different fire exposure can be assessed based on Eq (2). Based on the prescriptions in the Eurocode [15], the mean fire load density for an office and residential building corresponds to 420 MJ/m<sup>2</sup> and 780 MJ/m<sup>2</sup>, respectively. The maximum temperatures for these fire loads are 380 and 480 °C (for  $O = 0.04 \text{ m}^{1/2}$  and  $a = 20 \text{ mm}$ ) and the corresponding moment capacity can be calculated as 59 kN-m (same as design) and 45 kN-m, respectively, considering Eurocode [20] strength reduction factors.

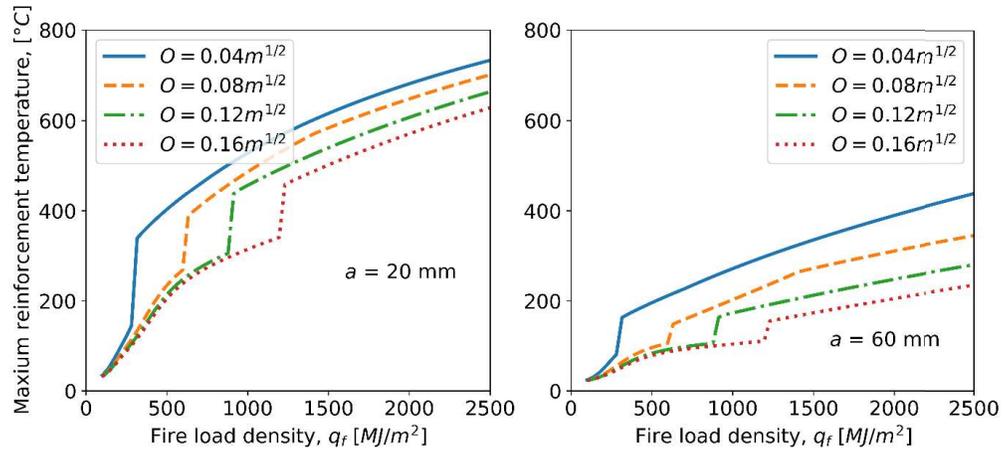


Figure 1. Maximum temperature of reinforcing bars of RC slab for parametric fire exposure, considering reinforcement axis distance,  $a$  of 20 and 60 mm.

## 4 DESIGN PARAMETERS AND OBJECTIVES FOR RC SLAB OPTIMIZATION

### 4.1 Definition of design parameters and objectives

For ambient design conditions, Eurocode [19] recommends a target reliability index of 3.8 for moderate consequences (50 year reference period). The parameters of bending moment capacity in Eq (2) can be adjusted to achieve this design target, e.g., by increasing the area of reinforcement  $A_s$ . The design target for fire exposure is however not clear. For fire design, Eurocode [20] tabulates a nominal 20 mm as the required reinforcement axis ( $a$ ) for RC slabs to achieve 60 minutes standard fire rating. This is a prescriptive value and the resulting safety level or economic optimality is unknown. Herein the optimum value of this parameter is decided through a risk-based design approach. As the reinforcement axis distance is varied, the ambient design capacity changes and there is a need to change the reinforcement area to maintain the ambient design capacity constant. These two variables ( $a$  and  $A_s$ ) are considered as design parameters in this study. Note also that the influence of other parameters on design moment capacity is relatively smaller. The influence of these parameters on ambient design moment capacity is shown in Figure 2.  $A_s$  significantly influences the slab's capacity, while the influence of  $a$  increases with an increase in  $A_s$ .  $a$  will however have a large influence on the slab capacity at elevated temperature, as can be deduced from the rebar temperatures in Figure 1.

Three costs are associated with evaluating the optimum design parameters for the slab: the investment cost (including its obsolescence value), the (lifetime) failure cost, and the environmental cost. The minimization of these costs allows designing the slab at minimum cost and therefore are considered as objective functions for design optimization of RC slab. The investment cost and the environmental cost are proportional to the amount of reinforcement in slab, while the failure cost is governed by the ultimate limit state and the structural failure probability. The impact of failure on the environmental cost is not considered at this point. The upfront justification is that the failure frequency should be so low that the environmental impact is limited. The Eurocode ambient design target [19] for reliability index is considered as constraint for the design optimization.

### 4.2 Investment cost

Out of the two design parameters, the investment cost is only associated with the increased reinforcement area of the RC slab. The reinforcement area of the slab with reinforcing bars of 10 mm diameter and spacing of 200 mm (centre to centre) is considered as reference case ( $A_s$  of 393 mm<sup>2</sup>). The maximum  $A_s$  considered for the optimization is 3930 mm<sup>2</sup>. Note that for this value, the section is not over-reinforced. Figure 3 shows the investment cost as a function of the reinforcement area in the slab. The cost is obtained from RSMMeans [21], a database for building and construction costs. The cost of reinforcing bars for the slab is

1.31 USD (\$)/lb (i.e., 2.89 \$/kg) for US national average. This cost includes the labour, material and overhead costs. For the reference case, the cost of reinforcement for the slab (per m<sup>2</sup>) is 2.89 (\$/kg) × 7850 (steel density in kg/m<sup>3</sup>) × 393/(106) (m<sup>2</sup>) = 8.92 \$. For the maximum limit of reinforcement area (i.e., 393 mm<sup>2</sup>), the cost could increase to 89.2 \$/m<sup>2</sup>. For slab of size (6m × 4 m), the cost is 2141 \$, an increase of 1927 \$ over the reference case. This cost corresponds to a single slab in the building. Considering all the slabs in the building, the cost would be large. The cost has been evaluated considering the slab size of 6 m × 4 m. In this study, the investment cost includes the obsolescence value (lifetime). An obsolescence rate ( $\omega$ ) of 0.022/year [22] is considered for this. For lifetime evaluation of obsolescence cost, a discounting factor of 0.03/year (for societal stakeholders [23]) is applied. The total investment cost,  $C_I$  with obsolescence value can be written as:

$$C_I = C_{I,0} \left(1 + \frac{\omega}{\gamma}\right) \quad (3)$$

where,  $C_{I,0}$  is the investment cost for reinforcing bars in the slab.

### 4.3 Environmental cost

Environmentally friendly design solutions are increasingly preferred over cost-effective solutions [24] because of the increased concerns related to greenhouse emissions from the construction industry (amounting to about 12 % of total emissions in Western Europe). This study, therefore, considers reducing the environmental impact of structural design as one of the objectives, with CO<sub>2</sub> emissions as the environmental cost indicator [25]. The cost of added reinforcements for RC slab contributes to the additional CO<sub>2</sub> emission. This is estimated based on [24], where the total life-cycle CO<sub>2</sub> emission per kg of steel is estimated as 3.01 kg. Figure 3 shows the CO<sub>2</sub> emissions calculated for the increased reinforcement area of RC slab. The increased reinforcement area could cause 2000 kg CO<sub>2</sub> emission equivalent and thus there is a need to consider this cost, which in the reference case amounts to 220 kg (10 times lower).

### 4.4 Life-time failure cost

Failure cost minimization under fire-exposure is another objective considered in the design optimization of the RC slab. The evaluation of the life-time cost ( $D$ ) is risk-based and conditional on the fire exposure scenario, given by:

$$D = \frac{\lambda_{fi} P_{sf} \mu_D}{\gamma} \quad (4)$$

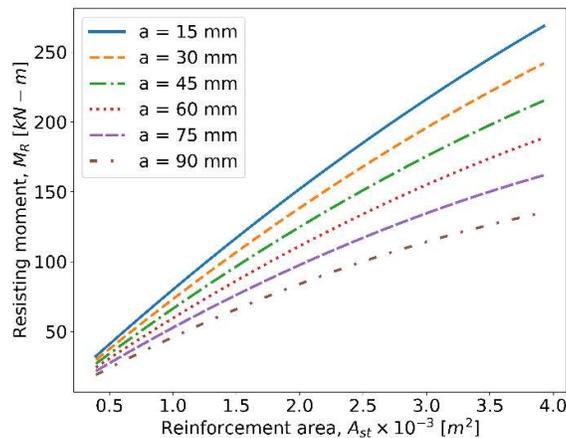


Figure 2 Ambient bending moment capacity of the RC slab as a function of reinforcement area and its axis position.

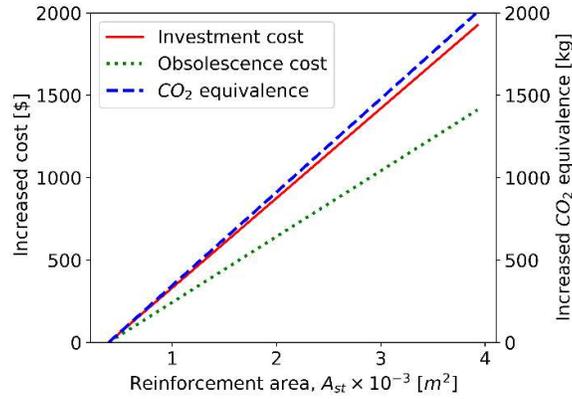


Figure 3. Investment cost, obsolescence cost, and CO<sub>2</sub> emission [kg] equivalent function of the reinforcement area of the slab. where,  $\lambda_{fi}$  is the yearly probability of a structurally significant fire,  $P_{sf}$  is the probability of structural failure given fire, and  $\mu_D$  is the mean failure cost for the given fire. The same discounting factor ( $\gamma$ ) as in the obsolescence life-time cost evaluation is taken here (0.03 for societal stakeholders). Based on [26], the annual fire occurrence frequency for office building is 0.00423, while it is 0.00151 for a residential building (united states). Probability of fire occurrence are reported values, while the failure normally occurs for structurally significant fires. The structurally significant fires are commonly estimated by considering successful early fire suppression because of the presence of active fire protection systems (reduction factor of 0.08 considered [27]) and other factors such as the efficiency of the fire and rescue service system (reduction factor of 0.10 considered [28]). The evaluation of the failure cost for the structure includes direct costs and indirect costs, and is normally evaluated as a factor ( $\xi$ ) of the initial construction cost ( $C_0$ ), as in Eq (5). Direct costs involve the costs from structural components, non-structural components, contents of the building, fatalities and injuries of civilians and fire-fighters, and can be statistically determined. Determination of indirect cost is challenging and often subjective.. In this study, the failure cost for the RC slab is evaluated as a factor of the initial structural cost. Based on RSMeans [21], the cost per square foot for an office building is ( $C_0$ ) 303 \$ psf (3261 \$/m<sup>2</sup>), while that of a multi-family dwelling (1-3) is 215 \$ psf (2314 \$/m<sup>2</sup>). These costs include the cost of both structural and non-structural components. The failure cost is evaluated considering the loss of entire compartment in the event of failure. Here, a failure cost factor of 100 is adopted for the evaluations and also a parametric study is conducted for this. This cost takes into account that the failure of a compartment slab is likely to result in extensive damages to the remainder of the building and inability to use a large part of the structure for a prolonged time. For office buildings a failure cost factor of 7 is commonly considered in earthquake engineering [29]. The cost factor considered here thus assumes that the local structural failure results in a loss of usability in an area with order of magnitude 15 times larger than the 24 m<sup>2</sup> compartment.

$$\mu_D = \xi C_0 \quad (5)$$

The structural failure probability is evaluated considering the exceedance of the bending moment capacity of the slab as a limit state. The limit state equation,  $Z$  can be written as:

$$Z = K_R M_{R,fi} - K_E (M_G + M_Q) \quad (6)$$

where,  $M_{R,fi}$  stands for bending moment capacity for fire exposure and  $M_G$  and  $M_Q$  are the moments from permanent and imposed loads, respectively.  $M_{R,fi}$  is evaluated based on Eq (2), while the characteristic moments due to loads are evaluated from the design moment of the considered slab, considering Eurocode safety factors.  $K_R$  and  $K_E$  are the model uncertainties for the capacity and load evaluations.

The characteristic moments due to loads are:

$$M_G = \frac{M_{Rd}}{\max\left\{\left(\gamma_G + \psi_0 \gamma_Q \frac{\chi}{1-\chi}\right); \left(\xi \gamma_G + \gamma_Q \frac{\chi}{1-\chi}\right)\right\}} \quad (7)$$

$$M_Q = \frac{\chi}{(1-\chi)} M_G \quad (8)$$

where,  $M_{Rd}$  refers to the design moment capacity of the slab at ambient conditions for the reference design (calculated as 59 kN-m).  $\gamma_G$  (1.35) and  $\gamma_Q$  (1.5) are the partial safety factors for the permanent and imposed load.  $\xi$  (0.85) is the reduction factor considering the unfavourable occurrence of permanent load.  $\psi_0$  for office building is commonly 0.7. The same value of  $\psi_0$  is considered for residential building as well. The load factor ( $\chi$ ) is assumed as 0.5 for the load moment evaluations.

The failure probability of the RC slab is evaluated based on Eq (9), where  $Z$  is the limit state equation. Table 1 lists the stochastic parameters for evaluating  $M_{R,fi}$  and the load moments ( $M_Q$  and  $M_G$ ). A Monte-Carlo approach is adopted for the evaluations.

$$P_{sf} = P[Z < 0] \quad (9)$$

$M_{R,fi}$  involves a thermo-mechanical analysis as discussed in Section 3.2. Note that in Table 2, the probabilistic model for strength retention factor for yield strength of reinforcing bars include the uncertainty in yield strength at 20°C.  $10^5$  MC (Monte Carlo) realizations are developed for the estimation of  $M_{R,fi}$ . For computational efficiency, an interpolation function (3-dimensional) is developed to evaluate the temperature of reinforcing bars, with axis distance ('a'), fire load density ( $q_f$ ) and compartment opening factor ( $O$ ) as input parameters. The evaluation for load moments is based on  $10^{10}$  MC realizations. Figure 4 shows the failure probability for the slab at different values of design parameters ('a' and  $A_s$ ). Two fire exposure scenarios are considered (i.e.,  $q_{f,nom}$  of 420 and 780 MJ/m<sup>2</sup>). From the figure, it can be concluded that the addition of reinforcing bars could reduce the failure probability of RC slab from a level of 1.0 to  $10^{-9}$ . Reinforcement axis distance changes the failure probability marginally in slabs with a lower reinforcement area, while significantly with a higher reinforcement area. Comparatively, fire exposure with 780 MJ/m<sup>2</sup> is considerably more critical than 420 MJ/m<sup>2</sup> for the RC slab when comparing failure probability.

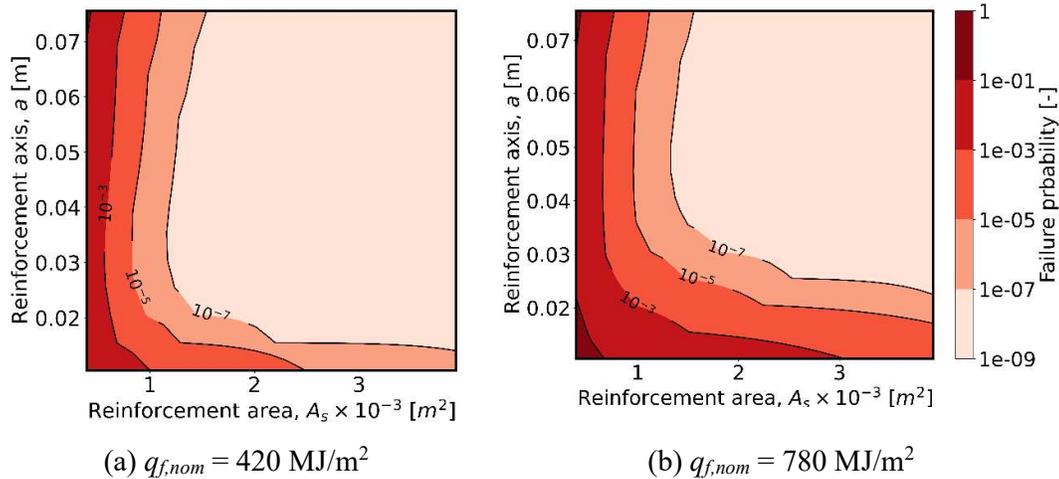


Figure 4 Failure probability of RC slab for fire exposure with fire load density of (a) 420 and (b) 780 MJ/m<sup>2</sup> at different values of reinforcement area and its axis position (constant compartment opening factor of  $O = 0.04 \text{ m}^{1/2}$ ).

Table 1 Stochastic parameters for probabilistic evaluation of moment capacity and the moments due to load for RC slab

Variables	Symbol [unit]	Distribution	Mean, $\mu$	SD, $\sigma$
<b>Moment capacity, <math>M_{R,f}</math></b>				
Slab thickness	$h$ [m]	Normal [30]	200	5
Concrete strength	$f_c$ (20°C) [MPa]	Lognormal [30]	42.9	6.4
Reinforcement yield strength retention factor	$k_{f,T}$ [-]	Temperature dependent lognormal model [31]		
Reinforcement axis	'a' [mm]	Beta(4,4) [30]	$a_{nom} + 5$ [12 – 80]	5
Reinforcement area	$A_s$ [mm <sup>2</sup> /m]	Normal [30]	$1.02 A_{s,nom}$	$0.02\mu_{A_s}$
Fire load density	$q_f$ [MJ/m <sup>2</sup> ]	Gumbel [15]	$q_{f,nom}$ [420 and 780]	$0.3\mu_{q_f,nom}$
Compartment opening factor	$O$ [m <sup>1/2</sup> ]	Deterministic	$O$ [0.04-0.16]	-
<b>Moment due to loads</b>				
Permanent load	$M_G$ [kN-m]	Normal [32]	$M_G$	$0.1 \mu_{M,G}$
Imposed load	$M_Q$ [kN-m]	Gamma [32]	$0.2 \times M_Q$	$0.95\mu_{M,Q}$
<b>Model uncertainties</b>				
capacity estimation	$K_R$ [-]	Lognormal [30]	1.1	0.11
Load estimation	$K_E$ [-]	Lognormal [32]	1	0.1

#### 4.5 Ambient design verification

In this study, the ambient reliability index is considered as a constraint so that the design of RC slab is optimized for fire exposure only. he considered ambient design target reliability in this study (3.8) is lower than the requirement in EN 1990:2002 since the load models applied are for a 1-year reference period. Figure 5 shows the ambient design target reliability evaluated at different values of the reinforcement area and its axis positions. In the Figure, with an increase in reinforcement area, the ambient capacity of the RC slab increases and thus also the corresponding reliability index. Conversely, an increase in axis distance (from the bottom face) reduces the reliability.

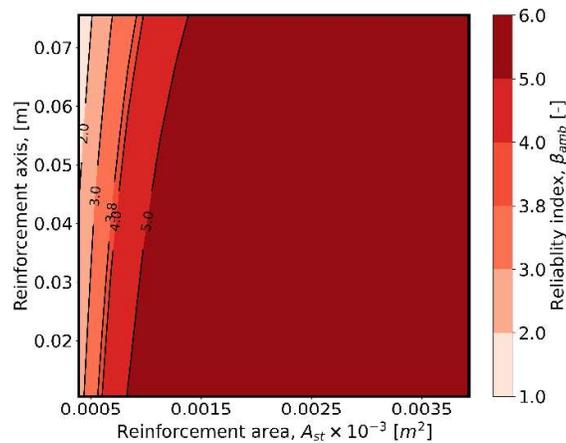


Figure 5 Ambient design target reliability index for RC slab at different values of design parameters.

## 5 MULTI-OBJECTIVE OPTIMIZATION FOR A RC SLAB EXPOSED TO FIRE

In the RC slab optimization, two design parameters, three design objectives, and one constraint have been identified (in Section 4). The NSGA-II algorithm is adopted for the optimization, where a population size of 100 for 60 generations, with 10 off-springs in each generation are used for the optimization input. Diverse Pareto-front solutions are obtained for these NSGA-II optimization parameters. The Pareto-front solutions can be seen in Figure 6b where the objective space is presented. The design space shows the points corresponding to the Pareto-front solutions of Figure 6a. The results are shown here only for RC slabs of a residential building occupancy, considering  $q_f$  of 780 MJ/m<sup>2</sup> and  $O$  of 0.04 m<sup>1/2</sup>. A failure cost factor ( $\xi$ ) equal to 100 times the initial construction cost is considered. The optimum values for  $A_s$  and  $a$  obtained are 583 mm<sup>2</sup>/m and 16 mm, respectively. These are shown by grey lines in the figure. The optimum value is obtained by evaluating the distance of the normalized Pareto-front solutions from an ideal solution where investment, failure and environmental cost have zero values. The Pareto-front point with the minimum distance is the optimum solution. The investment and the failure cost-points are normalized by a maximum value of these two costs for all Pareto-front points, whereas the environmental cost by maximum value of the environmental cost before evaluating the distances. There is a possibility to consider different (need-based) importance factor for the design objectives, for example, environmental costs could be more significant. Additionally, the environmental cost here is CO<sub>2</sub> emission based and could be transformed to an equivalent monetary value. For this situation, the normalization factor would be the maximum of all these three costs and the obtained optimum solution is economically optimum.

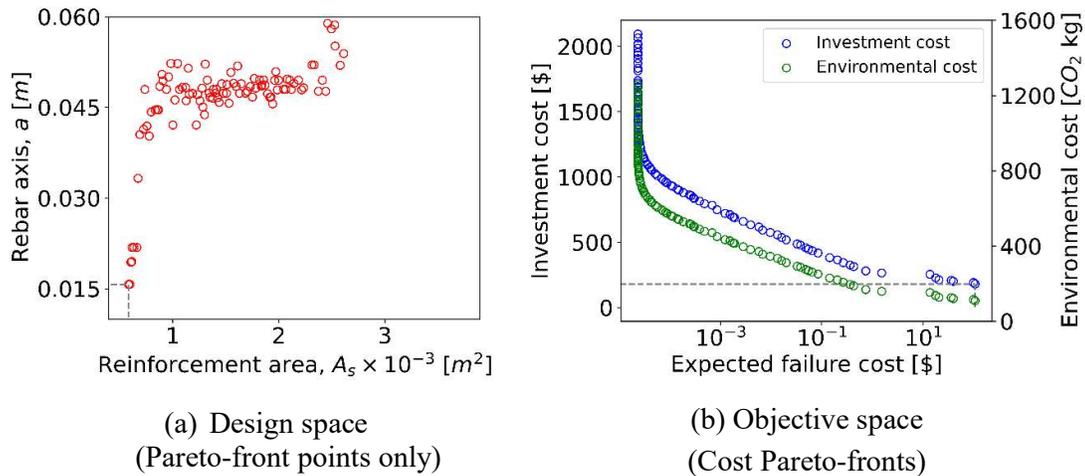


Figure 6 (a) Design space and (b) objective space for MOO of a RC slab, considering residential occupancy ( $q_f = 780$  MJ/m<sup>2</sup> and  $O = 0.04$  m<sup>1/2</sup>). The failure cost factor is 100 times of the initial structural cost

The determination of failure cost factor ( $\xi$ ) of a fire-exposed RC slab is problematic, owing to the uncertain costs of the contents inside the building, fatalities and injury to occupants and the associated indirect costs. The failure cost factor is thus considered as a variable in the study. Table 2 presents the optimum value of the design parameters at different failure cost factors. Also, the optimum objective function evaluations are shown in the Table. Results are shown for both the residential and office occupancy of the building. In the Table, the expected annual failure cost evaluated for lower  $\xi$  ( $< 100$ ) is smaller in comparison to the investment cost. This is because ambient design constraint ( $\beta_{amb}$ ) governs the optimum parameters for these cases. In Table 2, the optimum reinforcement area for  $\xi$  of 3 and 10 is 584 mm<sup>2</sup>, while reinforcement axis positions are 16 mm. The optimum 'a' here is based on failure cost minimization and therefore, the total cost value increases for both the increase or decrease in 'a'.

The considered slab in the study has reinforcement area of 785 mm<sup>2</sup> (10 rebars at 100 mm c/c). Based on [20] (see, Table 2), the axis positions are 20 mm and 40 mm for 1- and 2-hour of ISO fire-rating. When compared to the considered slab, the optimum design (with  $\xi$  of 2 and 10) has 25 % less reinforcement cost and the axis position of 16 mm is optimum. Further, considering the depth of slab as fixed, the higher axis positions recommendation in [20] for normal design condition (for 1- and 2-hour of ISO fire-rating)

accounts for increase in reinforcement area to 785 mm<sup>2</sup> and 896 mm<sup>2</sup> to maintain constant design bending moment. This shows the reinforcement cost to be 2.4% and 15 % lesser for MOO based optimum design . Also, the CO<sub>2</sub> emissions is 25 % (linear model considered) for the MOO design. The same observation can also be made for the residential building. In Table 2, the failure cost could go as high as 606 \$ for  $\xi$  of 10000. The failure cost can be high when a prescriptive approach is adopted, since 20 mm of cover is recommended by [20] for 1-hour of prescriptive fire resistance. Thus, MOO based designs leads to economical design solutions.

These design optimizations are presented here for a single RC slab member. If all of the slabs for a multi-story structure are considered, the overall cost reduction can be significant. In Table 2, we see how the reinforcement area and axis positions vary with the increased  $\xi$  and thus “building importance classes” can be defined for these different levels of  $\xi$  with target reliabilities, and associated “prescriptive” design provisions.

In this study, a MOO approach is demonstrated for a risk-based design of a RC slab and thus can be generalized for other structural members under fire-exposure. Currently, the presented MOO approach considers only two design parameters and three design objectives, but additional multiple objectives can be introduced in the same framework. Further, minimization of environmental damage is adopted as another dimension of the objective (which is seldom considered in design), leading to a more sustainable design. The MOO framework has environmental damage measured as CO<sub>2</sub> emissions and thus this framework has the possibility to consider diverse objective types and is not limited to a single objective measure (for example, monetary evaluations for failure and investment cost).

Table 2 Optimum design solutions for the RC slab at different failure cost factor values, considering office and residential occupancies.

Failure cost factor ( $\xi$ )	Optimum design parameters		Optimum costs		
	$A_s$ [mm <sup>2</sup> ]	a [mm]	Investment cost including obsolescence [\$]	Expected annual failure cost [\$]	Environmental cost [kg CO <sub>2</sub> ]
<b>Office building (<math>q_f = 420</math> MJ/m<sup>2</sup>)</b>					
<b>3</b>	584	16	180	13	108
<b>10</b>	584	16	180	42	108
<b>100</b>	617	22	212	82	127
<b>1000</b>	673	33	264	61	159
<b>10000</b>	673	33	264	606	159
<b>Residential building (<math>q_f = 780</math> MJ/m<sup>2</sup>)</b>					
<b>3</b>	584	16	180	3	108
<b>10</b>	584	16	180	11	108
<b>100</b>	584	16	180	107	108
<b>1000</b>	617	22	212	207	127
<b>10000</b>	673	33	264	153	159
<b>EN1992:1-2 [20] recommendation for ‘a’</b>					
<b>ISO Fire rating</b>	R30	R60	R90	R120	R180
<b>a [mm]</b>	10	20	30	40	55

## 6 CONCLUSIONS

Prescriptive design approach for uncommon structures may be inadequate, either from an economic or reliability standpoint. For these types of structures, a performance-based approach is recommended, with explicit consideration of the risk associated with the design. This study highlights the potential benefits of performing this assessment through a Multi-Objective Optimization (MOO) framework to achieve a safe and economical structural design.

A MOO framework is proposed, which uses a genetic algorithm for the optimization. The framework is applied to evaluate the lifetime optimum design for a RC slab exposed to natural fires, considering multiple

objectives, design parameters and design constraints, within a probabilistic framework. The results indicate that the optimum value of design parameter is a function of the fire exposure parameter and the failure cost factor. The study shows that MOO approach leads to economical design solutions. For the considered RC slab, a reduction of about 40 % in reinforcement cost and 25 % in environmental cost can be made in comparison to the prescriptive design of 1- and 2-hour fire resistance rating. In comparison to single-objective optimization, the MOO approach is flexible and allows different metrics for objective functions, for example, investment and failure costs are expressed here in dollars, while environmental costs as CO<sub>2</sub> emissions. In this study, a framework for MOO based design of fire exposed structures is demonstrated where multiple structural performance criteria and objectives can be ascertained. These risk-based design approaches can be considered for defining prescriptive design provisions.

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